[2][a] $\theta = \frac{\pi}{2}$: $(-r, -\theta)$

$$-r = \sin 2(-\theta) \quad 1\tfrac{1}{2}$$
$$-r = \sin(-2\theta)$$
$$-r = -\sin 2\theta \quad 1$$
$$r = \sin 2\theta \quad 1$$

POLAR AXIS : $(-r, \pi - \theta)$

$$-r = \sin 2(\pi - \theta) \quad 1\tfrac{1}{2}$$
$$-r = \sin(2\pi - 2\theta)$$
$$-r = \sin(-2\theta)$$
$$-r = -\sin 2\theta \quad 1$$
$$r = \sin 2\theta \quad 1$$

[b] $\sin 2\theta = 1 + \cos 2\theta$, $\theta \in [0, 2\pi]$

$2\sin\theta\cos\theta = 1 + 2\cos^2\theta - 1$ $3\frac{1}{2}$

$2\sin\theta\cos\theta - 2\cos^2\theta = 0$

$2\cos\theta(\sin\theta - \cos\theta) = 0$ $1\frac{1}{2}$

$2\cos\theta = 0$ $\qquad$ $\sin\theta - \cos\theta = 0$

$\cos\theta = 0$ $\qquad$ $\sin\theta = \cos\theta$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ $1\frac{1}{2}$ $\qquad$ $\tan\theta = 1$ 1

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$ $1\frac{1}{2}$

$\theta = \frac{\pi}{2} \longrightarrow \boxed{r = \sin 2\left(\frac{\pi}{2}\right) = 0} \longrightarrow \boxed{(x,y) = (0,0)}$

$\theta = \frac{3\pi}{2} \longrightarrow \boxed{r = \sin 2\left(\frac{3\pi}{2}\right) = 0}$ 1 $\longrightarrow \boxed{(x,y) = (0,0)}$ 1

$\theta = \frac{\pi}{4} \longrightarrow r = \sin 2\left(\frac{\pi}{4}\right) = 1$ 1 $\longrightarrow (x,y) = \left(1\cdot\cos\frac{\pi}{4}, 1\cdot\sin\frac{\pi}{4}\right)$

$= \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ $1\frac{1}{2}$

$\theta = \frac{5\pi}{4} \longrightarrow r = \sin 2\left(\frac{5\pi}{4}\right) = 1$ 1 $\longrightarrow (x,y) = \left(1\cdot\cos\frac{5\pi}{4}, 1\cdot\sin\frac{5\pi}{4}\right)$

$= \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ $1\frac{1}{2}$

[c] $\underline{\sin 2\theta = 0}$ $1\tfrac{1}{2}$ $\quad \theta \in [0, 2\pi] \longrightarrow 2\theta \in [0, 4\pi]$

$\underline{2\theta = 0, \pi, 2\pi, 3\pi, 4\pi}$ $1\tfrac{1}{2}$

$\underline{\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi}$ 1 $\longrightarrow (r, \theta) = \boxed{(0,0), (0,\tfrac{\pi}{2}), (0,\pi), (0,\tfrac{3\pi}{2}), (0,2\pi)}$ 1

$\underline{1 + \cos 2\theta = 0}$ $1\tfrac{1}{2}$

$\underline{\cos 2\theta = -1}$ 1

$\underline{2\theta = \pi, 3\pi}$ $1\tfrac{1}{2}$

$\underline{\theta = \frac{\pi}{2}, \frac{3\pi}{2}}$ 1 $\longrightarrow (r, \theta) = \boxed{(0, \tfrac{\pi}{2}), (0, \tfrac{3\pi}{2})}$ 1

$-r = \sin 2(\pi + \theta) \longrightarrow r = -\sin(2\pi + 2\theta) \longrightarrow r = -\sin 2\theta$

$3\tfrac{1}{2}$ $\underline{-\sin 2\theta = 1 + \cos 2\theta}, \ \theta \in [0, \pi]$

$1\tfrac{1}{2}$ $\underline{-2\sin\theta\cos\theta = 1 + 2\cos^2\theta - 1}$

$0 = 2\cos^2\theta + 2\sin\theta\cos\theta$

$1\tfrac{1}{2}$ $\underline{0 = 2\cos\theta(\cos\theta + \sin\theta)}$

$2\cos\theta = 0 \qquad\qquad \cos\theta + \sin\theta = 0$

$\cos\theta = 0 \qquad\qquad \sin\theta = -\cos\theta$

$\underline{\theta = \frac{\pi}{2}}$ 1 $\qquad\qquad \underline{\tan\theta = -1}$ 1

$\underline{\theta = \frac{3\pi}{4}}$ 1

ON $r = 1 + \cos 2\theta$

$\theta = \frac{\pi}{2} \rightarrow r = 1 + \cos 2(\tfrac{\pi}{2}) = 0 \longrightarrow (r, \theta) = \underline{(0, \tfrac{\pi}{2})}$

ON $r = \sin 2\theta$

$\theta = \frac{\pi}{2} + \pi = \frac{3\pi}{2} \longrightarrow r = \sin 2(\tfrac{3\pi}{2}) = 0 \longrightarrow (r, \theta) = \underline{(0, \tfrac{3\pi}{2})}$ 1

ON $r = 1 + \cos 2\theta$

$\theta = \frac{3\pi}{4} \longrightarrow r = 1 + \cos 2(\tfrac{3\pi}{4}) = 1 \longrightarrow (r, \theta) = \underline{(1, \tfrac{3\pi}{4})}$

ON $r = \sin 2\theta$

$\theta = \frac{3\pi}{4} + \pi = \frac{7\pi}{4} \longrightarrow r = \sin 2(\tfrac{7\pi}{4}) = -1 \longrightarrow (r, \theta) = \underline{(-1, \tfrac{7\pi}{4})}$ $1\tfrac{1}{2}$

$-r = 1 + \cos 2(\pi + \theta) \longrightarrow -r = 1 + \cos(2\pi + 2\theta)$

$\longrightarrow -r = 1 + \cos 2\theta \longrightarrow r = -1 - \cos 2\theta$

$3\frac{1}{2}$ $\underline{\sin 2\theta = -1 - \cos 2\theta}$, $\theta \in [0, \pi]$

$2\sin\theta\cos\theta = -1 - (2\cos^2\theta - 1)$

$2\sin\theta\cos\theta + 2\cos^2\theta = 0$ (SAME AS PREVIOUS EQ'N)

$1\frac{1}{2}$ $\underline{\theta = \frac{\pi}{2}, \frac{3\pi}{4}}$

ON $r = \sin 2\theta$

$\theta = \frac{\pi}{2} \longrightarrow r = \sin 2(\frac{\pi}{2}) = 0 \longrightarrow (r, \theta) = \underline{(0, \frac{\pi}{2})}$

ON $r = 1 + \cos 2\theta$

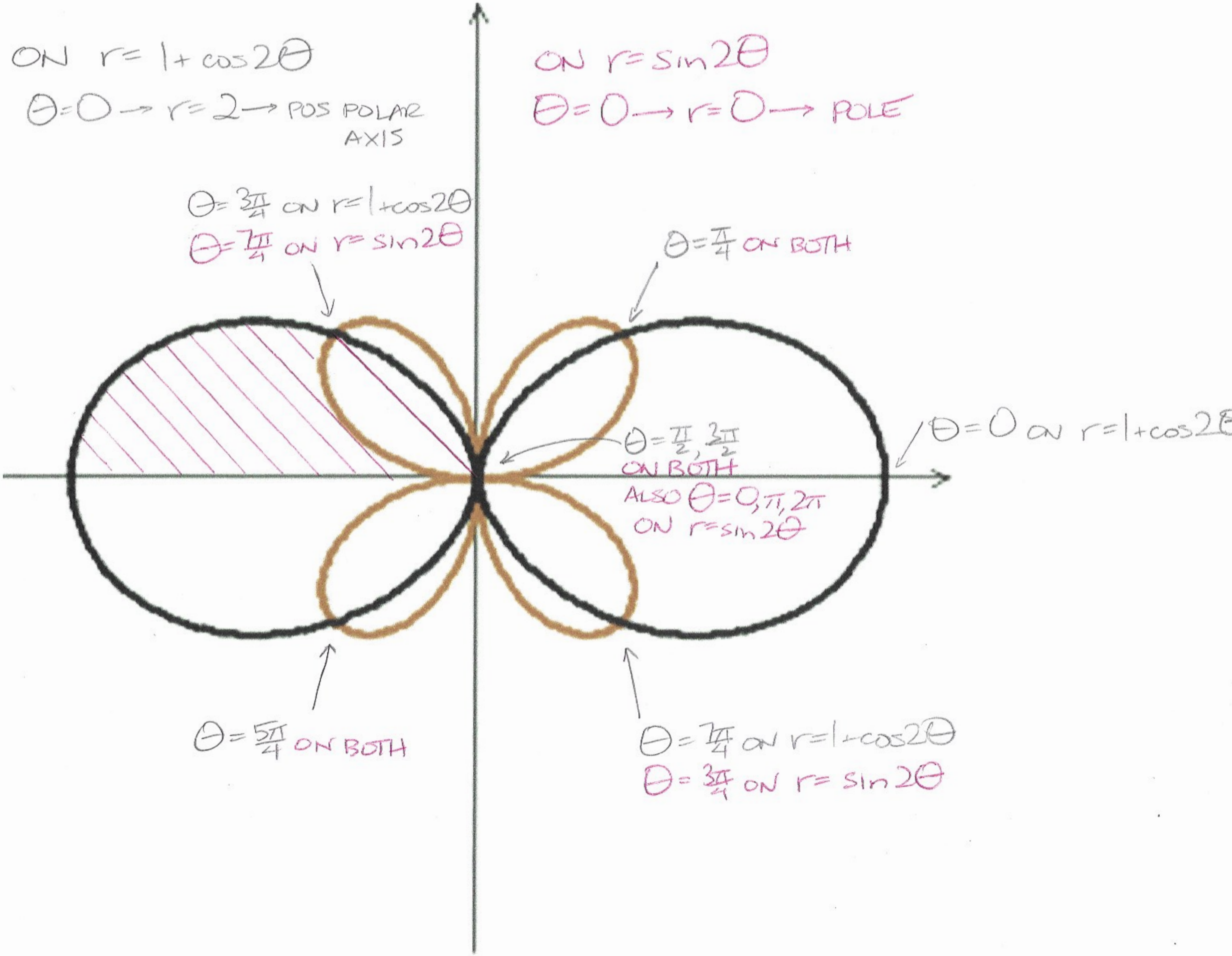
$\theta = \frac{\pi}{2} + \pi = \frac{3\pi}{2} \longrightarrow r = 1 + \cos 2(\frac{3\pi}{2}) = 0 \longrightarrow (r, \theta) = \underline{(0, \frac{3\pi}{2})}$ 1

ON $r = \sin 2\theta$

$\theta = \frac{3\pi}{4} \longrightarrow r = \sin 2(\frac{3\pi}{4}) = -1 \longrightarrow (r, \theta) = \underline{(-1, \frac{3\pi}{4})}$

ON $r = 1 + \cos 2\theta$

$\theta = \frac{3\pi}{4} + \pi = \frac{7\pi}{4} \longrightarrow r = 1 + \cos 2(\frac{7\pi}{4}) = 1 \longrightarrow (r, \theta) = \underline{(1, \frac{7\pi}{4})}$ $1\frac{1}{2}$



ON $r = 1 + \cos 2\theta$
$\theta = 0 \rightarrow r = 2 \rightarrow$ POS POLAR AXIS
ON $r = \sin 2\theta$
$\theta = 0 \rightarrow r = 0 \rightarrow$ POLE
$\theta = \frac{3\pi}{4}$ ON $r = 1 + \cos 2\theta$
$\theta = \frac{7\pi}{4}$ ON $r = \sin 2\theta$
$\theta = \frac{\pi}{4}$ ON BOTH
$\theta = 0$ ON $r = 1 + \cos 2\theta$
$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ ON BOTH
ALSO $\theta = 0, \pi, 2\pi$ ON $r = \sin 2\theta$
$\theta = \frac{5\pi}{4}$ ON BOTH
$\theta = \frac{7\pi}{4}$ ON $r = 1 + \cos 2\theta$
$\theta = \frac{3\pi}{4}$ ON $r = \sin 2\theta$

[d] $\underbrace{\boxed{\int_{\frac{3\pi}{4}}^{\pi}}}_{2} \underbrace{\frac{1}{2}(1+\cos 2\theta)^2}_{1\frac{1}{2}} d\theta - \underbrace{\boxed{\int_{\frac{7\pi}{4}}^{2\pi}}}_{2} \underbrace{\frac{1}{2}(\sin 2\theta)^2}_{1\frac{1}{2}} d\theta$

$$\int (1+\cos 2\theta)^2 \, d\theta = \int (\underbrace{1 + 2\cos 2\theta + \cos^2 2\theta}_{1\frac{1}{2}}) \, d\theta$$

$$= \int \left(1 + 2\cos 2\theta + \frac{1}{2}(1+\cos 4\theta)\right) d\theta$$

$$= \int \underbrace{\left(\frac{3}{2} + 2\cos 2\theta + \frac{1}{2}\cos 4\theta\right)}_{3\frac{1}{2}} d\theta$$

$$= \underbrace{\frac{3}{2}\theta + \sin 2\theta + \frac{1}{8}\sin 4\theta}_{3\frac{1}{2}} + C$$

$$\int (\sin 2\theta)^2 \, d\theta = \int \sin^2 2\theta \, d\theta$$

$$= \int \underbrace{\frac{1}{2}(1-\cos 4\theta)}_{1\frac{1}{2}} d\theta$$

$$= \frac{1}{2}\left(\theta - \frac{1}{4}\sin 4\theta\right) + C$$

$$= \underbrace{\frac{1}{2}\theta - \frac{1}{8}\sin 4\theta}_{1\frac{1}{2}} + C$$

$$\left(\tfrac{3}{2}\Theta + \sin 2\Theta + \tfrac{1}{8}\sin 4\Theta\right)\Big|_{\frac{3\pi}{4}}^{\pi}$$

$$= \tfrac{3}{2}\left(\pi - \tfrac{3\pi}{4}\right) + \left(\sin 2\pi - \sin \tfrac{3\pi}{2}\right) + \tfrac{1}{8}\left(\sin 4\pi - \sin 3\pi\right)$$

$$= \underline{\tfrac{3}{2}\,\tfrac{\pi}{4} + (0 - {}^{-}1) + \tfrac{1}{8}(0-0) = 1 + \tfrac{3\pi}{8}} \quad 1\tfrac{1}{2}$$

$$\left(\tfrac{1}{2}\Theta - \tfrac{1}{8}\sin 4\Theta\right)\Big|_{\frac{7\pi}{4}}^{2\pi}$$

$$= \tfrac{1}{2}\left(2\pi - \tfrac{7\pi}{4}\right) - \tfrac{1}{8}\left(\sin 8\pi - \sin 7\pi\right)$$

$$= \underline{\tfrac{1}{2}\,\tfrac{\pi}{4} - \tfrac{1}{8}(0-0) = \tfrac{\pi}{8}} \quad 2\tfrac{1}{2}$$

$$\text{AREA} = \tfrac{1}{2}\left(1 + \tfrac{3\pi}{8} - \tfrac{\pi}{8}\right) = \tfrac{1}{2}\left(1 + \tfrac{\pi}{4}\right) = \underline{\tfrac{1}{2} + \tfrac{\pi}{8}} \quad 1\tfrac{1}{2}$$

[e] $(r,\theta)=\left(-1,\frac{3\pi}{4}\right)$ on $r=\sin 2\theta$ FROM [c]

$$(x,y)=\left(-1\cos\frac{3\pi}{4},-1\sin\frac{3\pi}{4}\right)=\underline{\left(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)}\ 1\tfrac{1}{2}$$

$x=\sin 2\theta\cos\theta$

$y=\sin 2\theta\sin\theta$

$$\frac{dy}{dx}=\boxed{\frac{2\cos 2\theta\sin\theta+\sin 2\theta\cos\theta}{2\cos 2\theta\cos\theta-\sin 2\theta\sin\theta}}$$

8

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{3\pi}{4}}=\frac{2\cos\frac{3\pi}{2}\sin\frac{3\pi}{4}+\sin\frac{3\pi}{2}\cos\frac{3\pi}{4}}{2\cos\frac{3\pi}{2}\cos\frac{3\pi}{4}-\sin\frac{3\pi}{2}\sin\frac{3\pi}{4}}$$

$$=\boxed{\frac{2(0)\left(\frac{\sqrt{2}}{2}\right)+(-1)\left(-\frac{\sqrt{2}}{2}\right)}{2(0)\left(\frac{\sqrt{2}}{2}\right)-(-1)\left(\frac{\sqrt{2}}{2}\right)}=1}\ 1\tfrac{1}{2}$$

$$\underline{y+\frac{\sqrt{2}}{2}=x-\frac{\sqrt{2}}{2}}\ 1\tfrac{1}{2}\longrightarrow y=x-\sqrt{2}$$

[3][a] $C_1 : \underline{t = \frac{1-x}{2}}_{3} \longrightarrow y = 3\left(\frac{1-x}{2}\right) + 7 = \underline{-\frac{3}{2}x + \frac{17}{2}}_{3}$

$C_2 : \underline{e^t = x-1}_{2} \longrightarrow y = 6 - (x-1) = \underline{-x+7}_{2}$

$C_3 : \underline{t^2 = -x}_{2} \longrightarrow y = \underline{-x+7}_{2}$

[b] $C_3: y = t^2 + 7 = 0 \longrightarrow t \notin \mathbb{R}$

3

[c] $\boxed{\begin{aligned} 1-2t &= e^T+1 \\ 3t+7 &= 6-e^T \end{aligned}}$ ADD

4

$\boxed{\begin{aligned} 8+t &= 7 \\ t &= -1 \end{aligned}}$ $1\frac{1}{2}$ $\longrightarrow$ $\boxed{\begin{aligned} 1-2(-1) &= e^T+1 \\ 3 &= e^T+1 \\ T &= \ln 2 \end{aligned}}$ $1\frac{1}{2}$

[d] $C_2 : x = \underline{e^t + 1 > 1}$ 3

$C_3 : x = \underline{-t^2 \leq 0}$ 3

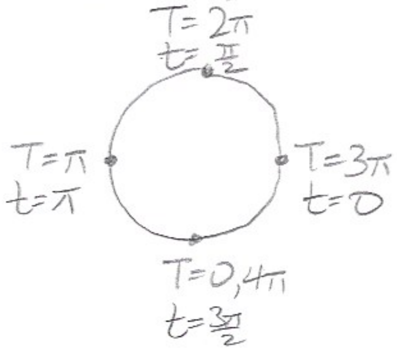
NO X-VALUES IN COMMON

[4] [a] CENTER (7, 0)

$x = 7 + 4\cos t$

$y = 0 + 4\sin t = 4\sin t$ $\quad t \in [0, 2\pi]$

ONCE AROUND
START @ RIGHT END
COUNTERCLOCKWISE



$t = mT + b$

$$\boxed{m = \frac{\Delta t}{\Delta T} = \frac{\pi - \frac{3\pi}{2}}{\pi - 0} = \frac{-\frac{\pi}{2}}{\pi} = -\frac{1}{2}}$$ 2

WHEN $T = 0$, $t = \frac{3\pi}{2} = b$

$\underline{t = -\frac{1}{2}T + \frac{3\pi}{2}}$ 6

$$\underline{x = 7 + 4\cos\left(-\tfrac{1}{2}T + \tfrac{3\pi}{2}\right)} = 7 + 4\left(\cos\left(-\tfrac{1}{2}T\right)\cancel{\cos\tfrac{3\pi}{2}}^{\,0} - \sin\left(-\tfrac{1}{2}T\right)\cancel{\sin\tfrac{3\pi}{2}}^{\,-1}\right)$$ 6

$$= \underline{7 - 4\sin\tfrac{1}{2}T}$$ 2

$$\underline{y = 4\sin\left(-\tfrac{1}{2}T + \tfrac{3\pi}{2}\right)} = 4\left(\sin\left(-\tfrac{1}{2}T\right)\cancel{\cos\tfrac{3\pi}{2}}^{\,0} + \cos\left(-\tfrac{1}{2}T\right)\cancel{\sin\tfrac{3\pi}{2}}^{\,-1}\right)$$ 6

$$= \underline{-4\cos\tfrac{1}{2}T}$$ 2

[b] $\underset{2}{\boxed{\int_{2\pi}^{4\pi}}} \underbrace{2\pi\left(7-4\sin\tfrac{1}{2}T\right)}_{2}\underbrace{\sqrt{\left(-2\cos\tfrac{1}{2}T\right)^2+\left(2\sin\tfrac{1}{2}T\right)^2}}_{4}\,dT$

$= \int_{2\pi}^{4\pi} 2\pi\left(7-4\sin\tfrac{1}{2}T\right)\underbrace{(2)}_{2}\,dT$

$= 4\pi\left(\underbrace{7T+8\cos\tfrac{1}{2}T}_{4}\right)\Big|_{2\pi}^{4\pi}$

$= 4\pi\left(7(4\pi-2\pi)+8(\cos 2\pi-\cos\pi)\right)$

$= 4\pi\left(14\pi+8(1--1)\right)$

$= \underbrace{4\pi(14\pi+16)}_{2}$

[c] $\underline{\int_0^{2\pi} 2\pi(7-4\sin\tfrac{1}{2}T)(2)\,dT}$ 2

$= \underline{4\pi\left(7T + 8\cos\tfrac{1}{2}T\right)\Big|_0^{2\pi}}$ 2

$= 4\pi\left(7(2\pi-0)+8(\cos\pi-\cos 0)\right)$

$= 4\pi(14\pi+8(-1-1))$

$= \underline{4\pi(14\pi-16)}$ 2

[d] $y = \underline{-4\cos\frac{1}{2}T = 2}$ 1, $T \in [2\pi, 3\pi] \longrightarrow \frac{1}{2}T \in [\pi, \frac{3\pi}{2}]$

$\underline{\cos\frac{1}{2}T = -\frac{1}{2}}$ 1

$\underline{\frac{1}{2}T = \frac{4\pi}{3}}$ 1

$\underline{T = \frac{8\pi}{3}}$ 1 $\longrightarrow x = 7 - 4\sin\frac{4\pi}{3} = \underline{7 + 2\sqrt{3}}$ 2

$$\frac{dy}{dx} = \boxed{\frac{2\sin\frac{1}{2}T}{-2\cos\frac{1}{2}T}}_{4} = -\tan\frac{1}{2}T$$

$$\left.\frac{dy}{dx}\right|_{T=\frac{8\pi}{3}} = -\tan\frac{4\pi}{3} = \underline{-\sqrt{3}}\ 2$$

$$\underline{y - 2 = -\sqrt{3}(x - 7 - 2\sqrt{3})}\ 2$$